

# PLTW Engineering Formula Sheet 2015 (v15.0)

## 1.0 Statistics

### Mean

$$\mu = \frac{\sum x_i}{N} \quad (1.1a) \qquad \bar{x} = \frac{\sum x_i}{n} \quad (1.1b)$$

$\mu$  = population mean  
 $\bar{x}$  = sample mean  
 $\sum x_i$  = sum of all data values ( $x_1, x_2, x_3, \dots$ )  
 $N$  = size of population  
 $n$  = size of sample

### Median

Place data in ascending order.  
 If  $N$  is odd, median = central value (1.2)  
 If  $N$  is even, median = mean of two central values  
 $N$  = size of population

### Range (1.5)

$$\text{Range} = x_{\max} - x_{\min} \quad (1.3)$$

$x_{\max}$  = maximum data value  
 $x_{\min}$  = minimum data value

### Mode

Place data in ascending order.  
 Mode = most frequently occurring value (1.4)

If two values occur with maximum frequency the data set is *bimodal*.  
 If three or more values occur with maximum frequency the data set is *multi-modal*.

### Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}} \quad (\text{Population}) \quad (1.5a)$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \quad (\text{Sample}) \quad (1.5b)$$

$\sigma$  = population standard deviation  
 $s$  = sample standard deviation  
 $x_i$  = individual data value ( $x_1, x_2, x_3, \dots$ )  
 $\mu$  = population mean  
 $\bar{x}$  = sample mean  
 $N$  = size of population  
 $n$  = size of sample

## 2.0 Probability

### Frequency

$$f_x = \frac{n_x}{n} \quad (2.1)$$

$f_x$  = relative frequency of outcome  $x$   
 $n_x$  = number of events with outcome  $x$   
 $n$  = total number of events

### Binomial Probability (order doesn't matter)

$$P_k = \frac{n!(p^k)(q^{n-k})}{k!(n-k)!} \quad (2.2)$$

$P_k$  = binomial probability of  $k$  successes in  $n$  trials  
 $p$  = probability of a success  
 $q = 1 - p$  = probability of failure  
 $k$  = number of successes  
 $n$  = number of trials

### Independent Events

$$P(A \text{ and } B \text{ and } C) = P_A P_B P_C \quad (2.3)$$

$P(A \text{ and } B \text{ and } C)$  = probability of independent events  $A$  and  $B$  and  $C$  occurring in sequence  
 $P_A$  = probability of event  $A$

### Mutually Exclusive Events

$$P(A \text{ or } B) = P_A + P_B \quad (2.4)$$

$P(A \text{ or } B)$  = probability of either mutually exclusive event  $A$  or  $B$  occurring in a trial  
 $P_A$  = probability of event  $A$

### Conditional Probability

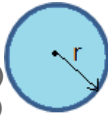
$$P(A|D) = \frac{P(A) \cdot P(D|A)}{P(A) \cdot P(D|A) + P(\sim A) \cdot P(D|\sim A)} \quad (2.5)$$

$P(A|D)$  = probability of event  $A$  given event  $D$   
 $P(A)$  = probability of event  $A$  occurring  
 $P(\sim A)$  = probability of event  $A$  not occurring  
 $P(D|\sim A)$  = probability of event  $D$  given event  $A$  did not occur

### 3.0 Plane Geometry

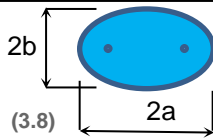
#### Circle

Circumference =  $2 \pi r$  (3.1)  
 Area =  $\pi r^2$  (3.2)



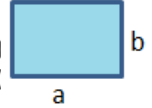
#### Ellipse

Area =  $\pi a b$  (3.8)



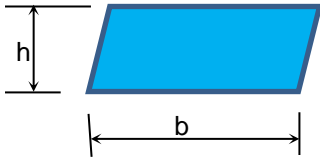
#### Rectangle

Perimeter =  $2a + 2b$  (3.9)  
 Area =  $ab$  (3.10)



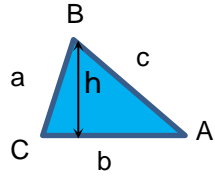
#### Parallelogram

Area =  $bh$  (3.3)



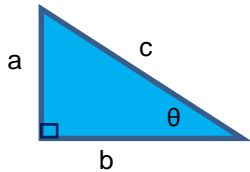
#### Triangle (3.6)

Area =  $\frac{1}{2} bh$  (3.11)  
 $a^2 = b^2 + c^2 - 2bc \cdot \cos \angle A$  (3.12)  
 $b^2 = a^2 + c^2 - 2ac \cdot \cos \angle B$  (3.13)  
 $c^2 = a^2 + b^2 - 2ab \cdot \cos \angle C$  (3.14)



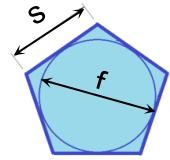
#### Right Triangle

$c^2 = a^2 + b^2$  (3.4)  
 $\sin \theta = \frac{a}{c}$  (3.5)  
 $\cos \theta = \frac{b}{c}$  (3.6)  
 $\tan \theta = \frac{a}{b}$  (3.7)



#### Regular Polygons

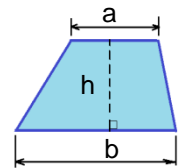
Area =  $n \frac{s(\frac{1}{2}f)}{2} = \frac{ns^2}{4 \tan(\frac{180}{n})}$  (3.15)



n = number of sides

#### Trapezoid

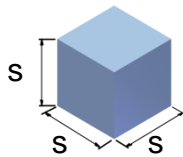
Area =  $\frac{1}{2}(a + b)h$  (3.16)



### 4.0 Solid Geometry

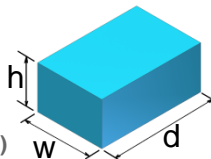
#### Cube

Volume =  $s^3$  (4.1)  
 Surface Area =  $6s^2$  (4.2)



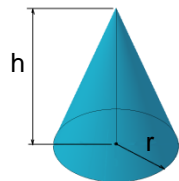
#### Rectangular Prism

Volume =  $w dh$  (4.3)  
 Surface Area =  $2(wd + wh + dh)$  (4.4)



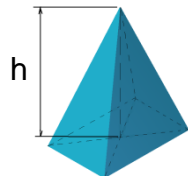
#### Right Circular Cone

Volume =  $\frac{\pi r^2 h}{3}$  (4.5)  
 Surface Area =  $\pi r \sqrt{r^2 + h^2}$  (4.6)



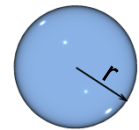
#### Pyramid

Volume =  $\frac{Ah}{3}$  (4.7)  
 A = area of base



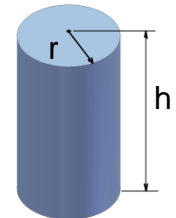
#### Sphere

Volume =  $\frac{4}{3} \pi r^3$  (4.8)  
 Surface Area =  $4 \pi r^2$  (4.9)



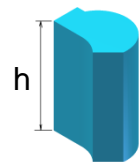
#### Cylinder

Volume =  $\pi r^2 h$  (4.10)  
 Surface Area =  $2 \pi r h + 2 \pi r^2$  (4.11)



#### Irregular Prism

Volume =  $Ah$  (4.12)  
 A = area of base



### 5.0 Constants

$g = 9.8 \text{ m/s}^2 = 32.27 \text{ ft/s}^2$   
 $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$   
 $\pi = 3.14159$

## 6.0 Conversions

### Mass/Weight (6.1)

1 kg	= 2.205 lb <sub>m</sub>
1 slug	= 32.2 lb <sub>m</sub>
1 ton	= 2000 lb
1 lb	= 16 oz

### Length (6.2)

1 m	= 3.28 ft
1 km	= 0.621 mi
1 in.	= 2.54 cm
1 mi	= 5280 ft
1 yd	= 3 ft

### Time (6.3)

1 d	= 24 h
1 h	= 60 min
1 min	= 60 s
1 yr	= 365 d

### Area (6.4)

1 acre	= 4047 m <sup>2</sup>
	= 43,560 ft <sup>2</sup>
	= 0.00156 mi <sup>2</sup>

### Volume (6.5)

1L	= 0.264 gal
	= 0.0353 ft <sup>3</sup>
	= 33.8 fl oz
1mL	= 1 cm <sup>3</sup> = 1 cc

### Temperature Unit Equivalents (6.6)

1 K	= 1 °C
	= 1.8 °F
	= 1.8 °R

### Force (6.7)

1 N	= 0.225 lb
1 kip	= 1,000 lb

### Pressure (6.8)

1 atm	= 1.01325 bar
	= 33.9 ft H <sub>2</sub> O
	= 29.92 in. Hg
	= 760 mm Hg
	= 101,325 Pa
	= 14.7 psi
1psi	= 2.31 ft of H <sub>2</sub> O

### Power (6.9)

1 W	= 3.412 Btu/h
	= 0.00134 hp
	= 14.34 cal/min
	= 0.7376 ft·lb <sub>f</sub> /s
1 hp	= 550 ft·lb/sec

### Energy (6.10)

1 J	= 0.239 cal
	= 9.48 x 10 <sup>-4</sup> Btu
	= 0.7376 ft·lb <sub>f</sub>
1kW h	= 3,600,000 J

### Rotational Speed (6.11)

1 Hz	= 2π rad/sec
	= 60 rpm

## 7.0 Defined Units

1 J	= 1 N·m
1 N	= 1 kg·m / s <sup>2</sup>
1 Pa	= 1 N / m <sup>2</sup>
1 V	= 1 W / A
1 W	= 1 J / s
1 Ω	= 1 V / A
1 Hz	= 1 s <sup>-1</sup>
1 F	= 1 A·s / V
1 H	= 1 V·s / V

## 8.0 SI Prefixes

### Numbers Less Than One

Power of 10	Prefix	Abbreviation
10 <sup>-1</sup>	deci-	d
10 <sup>-2</sup>	centi-	c
10 <sup>-3</sup>	milli-	m
10 <sup>-6</sup>	micro-	μ
10 <sup>-9</sup>	nano-	n
10 <sup>-12</sup>	pico-	p
10 <sup>-15</sup>	femto-	f
10 <sup>-18</sup>	atto-	a
10 <sup>-21</sup>	zepto-	z
10 <sup>-24</sup>	yocto-	y

### Numbers Greater Than One

Power of 10	Prefix	Abbreviation
10 <sup>1</sup>	deca-	da
10 <sup>2</sup>	hecto-	h
10 <sup>3</sup>	kilo-	k
10 <sup>6</sup>	Mega-	M
10 <sup>9</sup>	Giga-	G
10 <sup>12</sup>	Tera-	T
10 <sup>15</sup>	Peta-	P
10 <sup>18</sup>	Exa-	E
10 <sup>21</sup>	Zetta-	Z
10 <sup>24</sup>	Yotta-	Y

## 9.0 Equations

### Mass and Weight

$$m = VD_m \quad (9.1)$$

$$W = mg \quad (9.2)$$

$$W = VD_w \quad (9.3)$$

V = volume

D<sub>m</sub> = mass density

m = mass

D<sub>w</sub> = weight density

W = weight

g = acceleration due to gravity

### Temperature

$$T_K = T_C + 273 \quad (9.4)$$

$$T_R = T_F + 460 \quad (9.5)$$

$$T_F = \frac{9}{5} T_C + 32 \quad (9.6)$$

T<sub>K</sub> = temperature in Kelvin

T<sub>C</sub> = temperature in Celsius

T<sub>R</sub> = temperature in Rankin

T<sub>F</sub> = temperature in Fahrenheit

### Force and Moment

$$F = ma \quad (9.7a) \quad M = Fd_{\perp} \quad (9.7b)$$

F = force

m = mass

a = acceleration

M = moment

d<sub>⊥</sub> = perpendicular distance

### Equations of Static Equilibrium

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_P = 0 \quad (9.8)$$

F<sub>x</sub> = force in the x-direction

F<sub>y</sub> = force in the y-direction

M<sub>P</sub> = moment about point P

## 9.0 Equations (Continued)

### Energy: Work

$$W = F_{\parallel} \cdot d \quad (9.9)$$

W = work  
 $F_{\parallel}$  = force parallel to direction of displacement  
 d = displacement

### Power

$$P = \frac{E}{t} = \frac{W}{t} \quad (9.10)$$

$$P = \tau \omega \quad (9.11)$$

P = power  
 E = energy  
 W = work  
 t = time  
 $\tau$  = torque  
 $\omega$  = angular velocity

### Efficiency

$$\text{Efficiency (\%)} = \frac{P_{\text{out}}}{P_{\text{in}}} \cdot 100\% \quad (9.12)$$

$P_{\text{out}}$  = useful power output  
 $P_{\text{in}}$  = total power input

### Energy: Potential

$$U = mgh \quad (9.13)$$

U = potential energy  
 m = mass  
 g = acceleration due to gravity  
 h = height

### Energy: Kinetic

$$K = \frac{1}{2} mv^2 \quad (9.14)$$

K = kinetic energy  
 m = mass  
 v = velocity

### Energy: Thermal

$$\Delta Q = mc\Delta T \quad (9.15)$$

$\Delta Q$  = change in thermal energy  
 m = mass  
 c = specific heat  
 $\Delta T$  = change in temperature

### Fluid Mechanics

$$p = \frac{F}{A} \quad (9.16)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (\text{Charles' Law}) \quad (9.17)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad (\text{Gay-Lussanc's Law}) \quad (9.18)$$

$$p_1 V_1 = p_2 V_2 \quad (\text{Boyle's Law}) \quad (9.19)$$

$$Q = Av \quad (9.20)$$

$$A_1 v_1 = A_2 v_2 \quad (9.21)$$

$$P = Qp \quad (9.22)$$

absolute pressure = gauge pressure  
 + atmospheric pressure  $(9.23)$

p = absolute pressure  
 F = force  
 A = area  
 V = volume  
 T = absolute temperature  
 Q = flow rate  
 v = flow velocity  
 P = power

### Mechanics

$$\bar{s} = \frac{d}{t} \quad (9.24)$$

$$\bar{v} = \frac{\Delta d}{\Delta t} \quad (9.25)$$

$$a = \frac{v_f - v_i}{t} \quad (9.26)$$

$$X = \frac{v_i^2 \sin(2\theta)}{-g} \quad (9.27)$$

$$v = v_i + at \quad (9.28)$$

$$d = d_i + v_i t + \frac{1}{2} at^2 \quad (9.29)$$

$$v^2 = v_i^2 + 2a(d - d_i) \quad (9.30)$$

$$\tau = dF \sin \theta \quad (9.31)$$

$\bar{s}$  = average speed  
 $\bar{v}$  = average velocity  
 v = velocity  
 $v_i$  = initial velocity (t=0)  
 a = acceleration  
 X = range  
 t = time  
 $\Delta d$  = change in displacement  
 d = distance  
 $d_i$  = initial distance (t=0)  
 g = acceleration due to gravity  
 $\theta$  = angle  
 $\tau$  = torque  
 F = force

### Electricity

#### Ohm's Law

$$V = IR \quad (9.32)$$

$$P = IV \quad (9.33)$$

$$R_T (\text{series}) = R_1 + R_2 + \dots + R_n \quad (9.34)$$

$$R_T (\text{parallel}) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}} \quad (9.35)$$

#### Kirchhoff's Current Law

$$I_T = I_1 + I_2 + \dots + I_n$$

$$\text{or } I_T = \sum_{k=1}^n I_k \quad (9.36)$$

#### Kirchhoff's Voltage Law

$$V_T = V_1 + V_2 + \dots + V_n$$

$$\text{or } V_T = \sum_{k=1}^n V_k \quad (9.37)$$

V = voltage  
 $V_T$  = total voltage  
 I = current  
 $I_T$  = total current  
 R = resistance  
 $R_T$  = total resistance  
 P = power

### Thermodynamics

$$P = Q' = AU\Delta T \quad (9.38)$$

$$P = Q' = \frac{\Delta Q}{\Delta t} \quad (9.39)$$

$$U = \frac{1}{R} = \frac{k}{L} \quad (9.40)$$

$$P = \frac{kA\Delta T}{L} \quad (9.41)$$

$$A_1 v_1 = A_2 v_2 \quad (9.42)$$

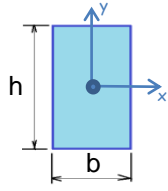
$$P_{\text{net}} = \sigma Ae(T_2^4 - T_1^4) \quad (9.43)$$

$$k = \frac{PL}{A\Delta T} \quad (9.44)$$

P = rate of heat transfer  
 Q = thermal energy  
 A = area of thermal conductivity  
 U = coefficient of heat conductivity (U-factor)  
 $\Delta T$  = change in temperature  
 $\Delta t$  = change in time  
 R = resistance to heat flow (R-value)  
 k = thermal conductivity  
 v = velocity  
 $P_{\text{net}}$  = net power radiated  
 $\sigma = 5.6696 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$   
 e = emissivity constant  
 L = thickness  
 $T_1, T_2$  = temperature at time 1, time 2

## 10.0 Section Properties

**Moment of Inertia**

$$I_{xx} = \frac{bh^3}{12} \quad (10.1)$$


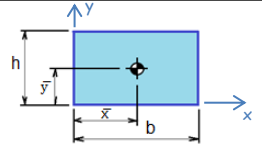
$I_{xx}$  = moment of inertia of a rectangular section about x axis

**Complex Shapes Centroid**

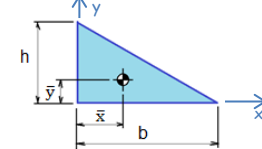
$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i} \quad \text{and} \quad \bar{y} = \frac{\sum y_i A_i}{\sum A_i} \quad (10.2)$$

$\bar{x}$  = x-distance to the centroid  
 $\bar{y}$  = y-distance to the centroid  
 $x_i$  = x distance to centroid of shape i  
 $y_i$  = y distance to centroid of shape i  
 $A_i$  = Area of shape i

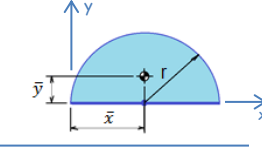
**Rectangle Centroid**

$$\bar{x} = \frac{b}{2} \quad \text{and} \quad \bar{y} = \frac{h}{2} \quad (10.3)$$


**Right Triangle Centroid**

$$\bar{x} = \frac{b}{3} \quad \text{and} \quad \bar{y} = \frac{h}{3} \quad (10.4)$$


**Semi-circle Centroid**

$$\bar{x} = r \quad \text{and} \quad \bar{y} = \frac{4r}{3\pi} \quad (10.5)$$


$\bar{x}$  = x-distance to the centroid  
 $\bar{y}$  = y-distance to the centroid

## 12.0 Structural Analysis

### 11.0 Material

**Stress (axial)**

$$\sigma = \frac{F}{A} \quad (11.1)$$

$\sigma$  = stress  
 $F$  = axial force  
 $A$  = cross-sectional area

**Strain (axial)**

$$\epsilon = \frac{\delta}{L_0} \quad (11.2)$$

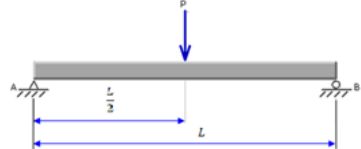
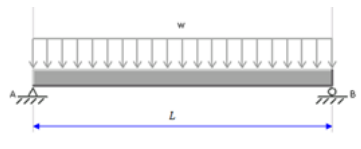
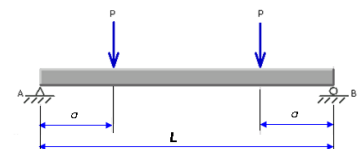

$\epsilon$  = strain  
 $L_0$  = original length  
 $\delta$  = change in length

**Modulus of Elasticity**

$$E = \frac{\sigma}{\epsilon} \quad (11.3)$$

$$E = \frac{(F_2 - F_1)L_0}{(\delta_2 - \delta_1)A} \quad (11.4)$$

$E$  = modulus of elasticity  
 $\sigma$  = stress  
 $\epsilon$  = strain  
 $A$  = cross-sectional area  
 $F$  = axial force  
 $\delta$  = deformation

<b>Beam Formulas</b>	
	<p><b>Reaction</b> <math>R_A = R_B = \frac{P}{2} \quad (12.1)</math></p> <p><b>Moment</b> <math>M_{max} = \frac{PL}{4}</math> (at point of load) (12.2)</p> <p><b>Deflection</b> <math>\Delta_{max} = \frac{PL^3}{48EI}</math> (at point of load) (12.3)</p>
	<p><b>Reaction</b> <math>R_A = R_B = \frac{\omega L}{2} \quad (12.4)</math></p> <p><b>Moment</b> <math>M_{max} = \frac{\omega L^2}{8}</math> (at center) (12.5)</p> <p><b>Deflection</b> <math>\Delta_{max} = \frac{5\omega L^4}{384EI}</math> (at center) (12.6)</p>
	<p><b>Reaction</b> <math>R_A = R_B = P \quad (12.7)</math></p> <p><b>Moment</b> <math>M_{max} = Pa \quad (12.8)</math></p> <p><b>Deflection</b> <math>\Delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2)</math> (at center) (12.9)</p>
	<p><b>Reaction</b> <math>R_A = \frac{Pb}{L}</math> and <math>R_B = \frac{Pa}{L} \quad (12.10)</math></p> <p><b>Moment</b> <math>M_{max} = \frac{Pab}{L}</math> (at Point of Load) (12.11)</p> <p><b>Deflection</b> <math>\Delta_{max} = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI}</math> (12.12)                      (at <math>x = \sqrt{\frac{a(a+2b)}{3}}</math> when <math>a &gt; b</math>)</p>

**Deformation: Axial**

$$\delta = \frac{FL_0}{AE} \quad (12.13)$$

$\delta$  = deformation  
 $F$  = axial force  
 $L_0$  = original length  
 $A$  = cross-sectional area  
 $E$  = modulus of elasticity

**Truss Analysis**

$$2J = M + R \quad (12.14)$$

$J$  = number of joints  
 $M$  = number of members  
 $R$  = number of reaction forces

# 13.0 Simple Machines

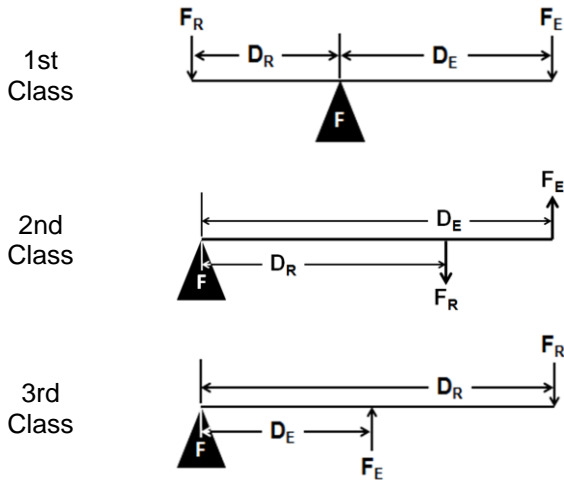
## Mechanical Advantage (MA)

$$IMA = \frac{D_E}{D_R} \quad (13.1) \quad AMA = \frac{F_R}{F_E} \quad (13.2)$$

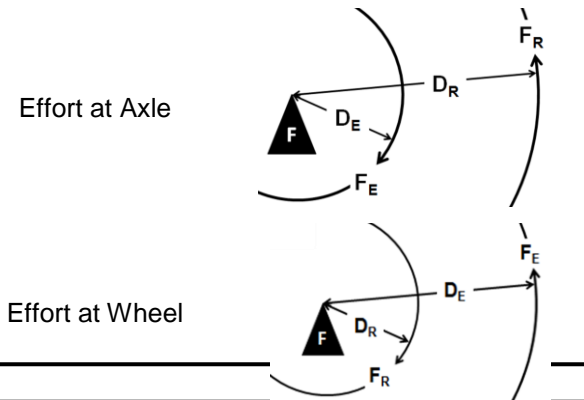
$$\% \text{ Efficiency} = \left( \frac{AMA}{IMA} \right) 100 \quad (13.3)$$

IMA = ideal mechanical advantage  
 AMA = actual mechanical advantage  
 $D_E$  = effort distance       $D_R$  = resistance distance  
 $F_E$  = effort force           $F_R$  = resistance force

## Lever



## Wheel and Axle



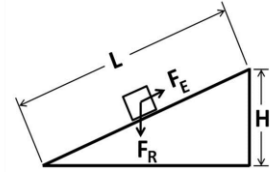
## Pulley Systems

IMA = total number of strands of a single string supporting the resistance (13.4)

$$IMA = \frac{D_E \text{ (string pulled)}}{D_R \text{ (resistance lifted)}} \quad (13.5)$$

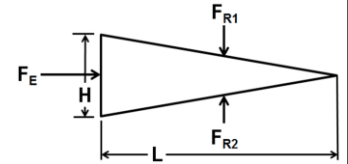
## Inclined Plane

$$IMA = \frac{L}{H} \quad (13.6)$$



## Wedge

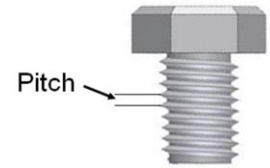
$$IMA = \frac{L}{H} \quad (13.7)$$



## Screw

$$IMA = \frac{C}{\text{Pitch}} \quad (13.8)$$

$$\text{Pitch} = \frac{1}{\text{TPI}} \quad (13.9)$$



C = circumference  
 r = radius  
 Pitch = distance between threads  
 TPI = threads per inch

## Compound Machines

$$MA_{\text{TOTAL}} = (MA_1) (MA_2) (MA_3) \dots \quad (13.10)$$

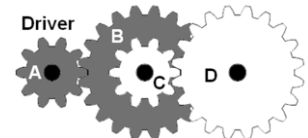
## Gears; Sprockets with Chains; and Pulleys with Belts Ratios

$$GR = \frac{N_{\text{out}}}{N_{\text{in}}} = \frac{d_{\text{out}}}{d_{\text{in}}} = \frac{\omega_{\text{in}}}{\omega_{\text{out}}} = \frac{\tau_{\text{out}}}{\tau_{\text{in}}} \quad (13.11)$$

$$\frac{d_{\text{out}}}{d_{\text{in}}} = \frac{\omega_{\text{in}}}{\omega_{\text{out}}} = \frac{\tau_{\text{out}}}{\tau_{\text{in}}} \text{ (pulleys)} \quad (13.12)$$

## Compound Gears

$$GR_{\text{TOTAL}} = \left( \frac{B}{A} \right) \left( \frac{D}{C} \right) \quad (13.13)$$



GR = gear ratio  
 $\omega_{\text{in}}$  = angular velocity - driver  
 $\omega_{\text{out}}$  = angular velocity - driven  
 $N_{\text{in}}$  = number of teeth - driver  
 $N_{\text{out}}$  = number of teeth - driven  
 $d_{\text{in}}$  = diameter - driver  
 $d_{\text{out}}$  = diameter - driven  
 $\tau_{\text{in}}$  = torque - driver  
 $\tau_{\text{out}}$  = torque - driven

## 14.0 Structural Design

### Steel Beam Design: Shear

$$V_a \leq \frac{V_n}{\Omega_v} \quad (14.1)$$

$$V_n = 0.6F_y A_w \quad (14.2)$$

$V_a$  = internal shear force

$V_n$  = nominal shear strength

$\Omega_v = 1.5$  = factor of safety for shear

$F_y$  = yield stress

$A_w$  = area of web

$\frac{V_n}{\Omega_v}$  = allowable shear strength

### Steel Beam Design: Moment

$$M_a \leq \frac{M_n}{\Omega_b} \quad (14.3)$$

$$M_n = F_y Z_x \quad (14.4)$$

$M_a$  = internal bending moment

$M_n$  = nominal moment strength

$\Omega_b = 1.67$  = factor of safety for bending moment

$F_y$  = yield stress

$Z_x$  = plastic section modulus about neutral axis

$\frac{M_n}{\Omega_b}$  = allowable bending strength

### Spread Footing Design

$$q_{\text{net}} = q_{\text{allowable}} - p_{\text{footing}} \quad (14.5)$$

$$p_{\text{footing}} = t_{\text{footing}} \cdot 150 \frac{\text{lb}}{\text{ft}^3} \quad (14.6)$$

$$q = \frac{P}{A} \quad (14.7)$$

$q_{\text{net}}$  = net allowable soil bearing pressure

$q_{\text{allowable}}$  = total allowable soil bearing pressure

$p_{\text{footing}}$  = soil bearing pressure due to footing weight

$t_{\text{footing}}$  = thickness of footing

$q$  = soil bearing pressure

$P$  = column load applied

$A$  = area of footing

## 15.0 Storm Water Runoff

### Storm Water Drainage

$$Q = C_f C_i A \quad (15.1)$$

$$C_c = \frac{C_1 A_1 + C_2 A_2 + \dots}{A_1 + A_2 + \dots} \quad (15.2)$$

$Q$  = peak storm water runoff rate ( $\text{ft}^3/\text{s}$ )

$C_f$  = runoff coefficient adjustment factor

$C$  = runoff coefficient

$i$  = rainfall intensity (in./h)

$A$  = drainage area (acres)

#### Runoff Coefficient Adjustment Factor

Return Period	$C_f$
1, 2, 5, 10	1.0
25	1.1
50	1.2
100	1.25

### Rational Method Runoff Coefficients

#### Categorized by Surface

Forested	0.059—0.2
Asphalt	0.7—0.95
Brick	0.7—0.85
Concrete	0.8—0.95
Shingle roof	0.75—0.95

#### Lawns, well drained (sandy soil)

Up to 2% slope	0.05—0.1
2% to 7% slope	0.10—0.15
Over 7% slope	0.15—0.2

#### Lawns, poor drainage (clay soil)

Up to 2% slope	0.13—0.17
2% to 7% slope	0.18—0.22
Over 7% slope	0.25—0.35

#### Driveways,

	0.75—0.85
--	-----------

#### Categorized by Use

Farmland	0.05—0.3
Pasture	0.05—0.3
Unimproved	0.1—0.3
Parks	0.1—0.25
Cemeteries	0.1—0.25
Railroad yard	0.2—0.40
Playgrounds	0.2—0.35

#### Business Districts

Neighborhood	0.5—0.7
City (downtown)	0.7—0.95
Residential	
Single-family	0.3—0.5
Multi-plexes,	0.4—0.6
Multi-plexes,	0.6—0.75
Suburban	0.25—0.4
Apartments,	0.5—0.7

#### Industrial

Light	0.5—0.8
Heavy	0.6—0.9

## 16.0 Water Supply

### Hazen-Williams Formula

$$h_f = \frac{10.44 L Q^{1.85}}{C^{1.85} d^{4.8655}} \quad (16.1)$$

$h_f$  = head loss due to friction (ft of  $\text{H}_2\text{O}$ )

$L$  = length of pipe (ft)

$Q$  = water flow rate (gpm)

$C$  = Hazen-Williams constant

$d$  = diameter of pipe (in.)

### Dynamic Head

dynamic head = static head – head loss (16.2)

static head = change in elevation between source and discharge

## 17.0 Heat Loss/Gain

### Heat Loss/Gain

$$Q' = AU\Delta T \quad (17.1)$$

$$U = \frac{1}{R} \quad (17.2)$$

$Q$  = thermal energy

$A$  = area of thermal conductivity

$U$  = coefficient of heat conductivity (U-factor)

$\Delta T$  = change in temperature

$R$  = resistance to heat flow (R-value)



## 18.0 Hazen-Williams Constants

Pipe Material	Typical Range	Clean, New Pipe	Typical Design Value
Cast Iron and Wrought Iron	80 - 150	130	100
Copper, Glass or Brass	120 - 150	140	130
Cement lined Steel or Iron		150	140
Plastic PVC or ABS	120 - 150	140	130
Steel, welded and seamless or interior riveted	80-150	140	100

## 19.0 Equivalent Length of (Generic) Fittings

Screwed Fittings		Pipe Size										
		1/4	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	4
Elbows	Regular 90 degree	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
	Long radius 90 degree	1.5	2.0	2.2	2.3	2.7	3.2	3.4	3.6	3.6	4.0	4.6
	Regular 45 degree	0.3	0.5	0.7	0.9	1.3	1.7	2.1	2.7	3.2	4.0	5.5
Tees	Line Flow	0.8	1.2	1.7	2.4	3.2	4.6	5.6	7.7	9.3	12.0	17.0
	Branch Flow	2.4	3.5	4.2	5.3	6.6	8.7	9.9	12.0	13.0	17.0	21.0
Return Bends	Regular 180 degree	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
Valves	Globe	21.0	22.0	22.0	24.0	29.0	37.0	42.0	54.0	62.0	79.0	110.0
	Gate	0.3	0.5	0.6	0.7	0.8	1.1	1.2	1.5	1.7	1.9	2.5
	Angle	12.8	15.0	15.0	15.0	17.0	18.0	18.0	18.0	18.0	18.0	18.0
	Swing Check	7.2	7.3	8.0	8.8	11.0	13.0	15.0	19.0	22.0	27.0	38.0
Strainer		4.6	5.0	6.6	7.7	18.0	20.0	27.0	29.0	34.0	42.0	

Flanged Fittings		Pipe Size																
		1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	4	5	6	8	10	12	14	16	18
Elbows	Regular 90 degree	0.9	1.2	1.6	2.1	2.4	3.1	3.6	4.4	5.9	7.3	8.9	12.0	14.0	17.0	18.0	21.0	23.0
	Long radius 90 degree	1.1	1.3	1.6	2.0	2.3	2.7	2.7	3.4	4.2	5.0	5.7	7.0	8.0	9.0	9.4	10.0	11.0
	Regular 45 degree	0.5	0.6	0.8	1.1	1.3	1.7	2.0	2.5	3.5	4.5	5.6	7.7	9.0	11.0	13.0	15.0	16.0
Tees	Line Flow	0.7	0.8	1.0	1.3	1.5	1.8	1.9	2.2	2.8	3.3	3.8	4.7	5.2	6.0	6.4	7.2	7.6
	Branch Flow	2.0	2.6	3.3	4.4	5.2	6.6	7.5	9.4	12.0	15.0	18.0	24.0	30.0	34.0	37.0	43.0	47.0
Return Bends	Regular 180 degree	0.9	1.2	1.6	2.1	2.4	3.1	3.6	4.4	5.9	7.3	8.9	12.0	14.0	17.0	18.0	21.0	23.0
	Long radius 180 degree	1.1	1.3	1.6	2.0	2.3	2.7	2.9	3.4	4.2	5.0	5.7	7.0	8.0	9.0	9.4	10.0	11.0
Valves	Globe	38.0	40.0	45.0	54.0	59.0	70.0	77.0	94.0	120.0	150.0	190.0	260.0	310.0	390.0			
	Gate						2.6	2.7	2.8	2.9	3.1	3.2	3.2	3.2	3.2	3.2	3.2	3.2
	Angle	15.0	15.0	17.0	18.0	18.0	21.0	22.0	285.0	38.0	50.0	63.0	90.0	120.0	140.0	160.0	190.0	210.0
	Swing Check	3.8	5.3	7.2	10.0	12.0	17.0	21.0	27.0	38.0	50.0	63.0	90.0	120.0	140.0			



## 20.0 555 Timer Design

$$T = 0.693 (R_A + 2R_B)C \quad (20.1)$$

$$f = \frac{1}{T} \quad (20.2)$$

$$\text{duty-cycle} = \frac{(R_A + R_B)}{(R_A + 2R_B)} \cdot 100\% \quad (20.3)$$

T = period

f = frequency

R<sub>A</sub> = resistance A

R<sub>B</sub> = resistance B

C = capacitance

## 21.B Resistor Color Code

	1 <sup>st</sup> Band	2 <sup>nd</sup> Band	Multiplier	Tolerance
NONE				20%
Silver			0.01	10%
Gold			0.1	5%
Black	0	0	1	
Brown	1	1	10	
Red	2	2	100	
Orange	3	3	1K	
Yellow	4	4	10K	
Green	5	5	100K	
Blue	6	6	1M	
Violet	7	7	10M	
Gray	8	8	100M	
White	9	9	1000M	

## 21.C Capacitor Code

Code	Tolerance
A	±0.05%
B	±0.1%
C	±0.25%
D	±0.5%
F	±1%
G	±2%
J	±5%
K	±10%
M or NONE	±20%
N	±30%
Q	-10%, +30%
S	-20%, +50%
T	-10%, +50%
Z	-20%, +80%

## 22.0 Speeds and Feeds

$$N = \frac{CS \left( \frac{12 \text{ in.}}{\text{ft}} \right)}{\pi d} \quad (22.1)$$

$$f_m = f_t \cdot n_t \cdot N \quad (22.2)$$

Plunge Rate =  $\frac{1}{2} \cdot f_m$

N = spindle speed (rpm)

CS = cutting speed (ft/min)

d = diameter (in.)

f<sub>m</sub> = feed rate (in./min)

f<sub>t</sub> = feed (in./tooth/rev)

n<sub>t</sub> = number of teeth

## 21.A Boolean Algebra

### Boolean Theorems

$$X \cdot 0 = 0 \quad (21.1)$$

$$X \cdot 1 = X \quad (21.2)$$

$$X \cdot X = X \quad (21.3)$$

$$X \cdot \bar{X} = 0 \quad (21.4)$$

$$X + 0 = X \quad (21.5)$$

$$X + 1 = 1 \quad (21.6)$$

$$X + X = X \quad (21.7)$$

$$X + \bar{X} = 1 \quad (21.8)$$

$$\bar{\bar{X}} = X \quad (21.9)$$

### Consensus Theorems

$$X + \bar{X}Y = X + Y \quad (21.16)$$

$$X + \bar{X}\bar{Y} = X + \bar{Y} \quad (21.17)$$

$$\bar{X} + XY = \bar{X} + Y \quad (21.18)$$

$$\bar{X} + X\bar{Y} = \bar{X} + \bar{Y} \quad (21.19)$$

### DeMorgan's Theorems

$$\overline{XY} = \bar{X} + \bar{Y} \quad (21.20)$$

$$\overline{X+Y} = \bar{X} \cdot \bar{Y} \quad (21.21)$$

### Commutative Law

$$X \cdot Y = Y \cdot X \quad (21.10)$$

$$X + Y = Y + X \quad (21.11)$$

### Associative Law

$$X(YZ) = (XY)Z \quad (21.12)$$

$$X + (Y + Z) = (X + Y) + Z \quad (21.13)$$

### Distributive Law

$$X(Y+Z) = XY + XZ \quad (21.14)$$

$$(X+Y)(W+Z) = XW+XZ+YW+YZ \quad (21.15)$$

## 23.0 G&M Codes

G00 = Rapid Traverse	(23.1)
G01 = Straight Line Interpolation	(23.2)
G02 = Circular Interpolation (clockwise)	(23.3)
G03 = Circular Interpolation (c-clockwise)	(23.4)
G04 = Dwell (wait)	(23.5)
G05 = Pause for user intervention	(23.6)
G20 = Inch programming units	(23.7)
G21 = Millimeter programming units	(23.8)
G80 = Canned cycle cancel	(23.9)
G81 = Drilling cycle	(23.10)
G82 = Drilling cycle with dwell	(23.11)
G90 = Absolute Coordinates	(23.12)
G91 = Relative Coordinates	(23.13)
M00 = Pause	(23.14)
M01 = Optional stop	(23.15)
M02 = End of program	(23.16)
M03 = Spindle on	(23.17)
M05 = Spindle off	(23.18)
M06 = Tool change	(23.19)
M08 = Accessory # 1 on	(23.20)
M09 = Accessory # 1 off	(23.21)
M10 = Accessory # 2 on	(23.22)
M11 = Accessory # 2 off	(23.23)
M30 = Program end and reset	(23.24)
M47 = Rewind	(23.25)

## 24.0 Aerospace

### Forces of Flight

$$C_D = \frac{2D}{A\rho v^2} \quad (24.1)$$

$$R_e = \frac{\rho v l}{\mu} \quad (24.2)$$

$$C_L = \frac{2L}{A\rho v^2} \quad (24.3)$$

$$M = Fd \quad (24.4)$$

$C_L$  = coefficient of lift  
 $C_D$  = coefficient of drag  
 $L$  = lift  
 $D$  = drag  
 $A$  = wing area  
 $\rho$  = density  
 $R_e$  = Reynolds number  
 $v$  = velocity  
 $l$  = length of fluid travel  
 $\mu$  = fluid viscosity  
 $F$  = force  
 $m$  = mass  
 $g$  = acceleration due to gravity  
 $M$  = moment  
 $d$  = moment arm (distance from datum perpendicular to F)

### Propulsion

$$F_N = W(v_j - v_o) \quad (24.5)$$

$$I = F_{ave}\Delta t \quad (24.6)$$

$$F_{net} = F_{avg} - F_g \quad (24.7)$$

$$a = \frac{v_f}{\Delta t} \quad (24.8)$$

$F_N$  = net thrust  
 $W$  = air mass flow  
 $v_o$  = flight velocity  
 $v_j$  = jet velocity  
 $I$  = total impulse  
 $F_{ave}$  = average thrust force  
 $\Delta t$  = change in time (thrust duration)  
 $F_{net}$  = net force  
 $F_{avg}$  = average force  
 $F_g$  = force of gravity  
 $v_f$  = final velocity  
 $a$  = acceleration  
 $\Delta t$  = change in time (thrust duration)

**NOTE:  $F_{ave}$  and  $F_{avg}$  are easily confused.**

### Energy

$$K = \frac{1}{2}mv^2 \quad (24.9)$$

$$U = \frac{-GMm}{R} \quad (24.10)$$

$$E = U + K = -\frac{GMm}{2R} \quad (24.11)$$

$$G = 6.67 \times 10^{-11} \frac{m^3}{kg \times s^2} \quad (24.12)$$

$K$  = kinetic energy  
 $m$  = mass  
 $v$  = velocity  
 $U$  = gravitational potential energy  
 $G$  = universal gravitation constant  
 $M$  = mass of central body  
 $m$  = mass of orbiting object  
 $R$  = Distance center main body to center of orbiting object  
 $E$  = Total Energy of an orbit

### Orbital Mechanics

$$e = \sqrt{1 - \frac{b^2}{a^2}} \quad (24.13)$$

$$T = 2\pi \frac{a^{\frac{3}{2}}}{\sqrt{\mu}} = 2\pi \frac{a^{\frac{3}{2}}}{\sqrt{GM}} \quad (24.14)$$

$$F = \frac{GMm}{r^2} \quad (24.15)$$

$e$  = eccentricity  
 $b$  = semi-minor axis  
 $a$  = semi-major axis  
 $T$  = orbital period  
 $a$  = semi-major axis  
 $\mu$  = gravitational parameter  
 $F$  = force of gravity between two bodies  
 $G$  = universal gravitation constant  
 $M$  = mass of central body  
 $m$  = mass of orbiting object  
 $r$  = distance between center of two objects

### Bernoulli's Law

$$\left(P_s + \frac{\rho v^2}{2}\right)_1 = \left(P_s + \frac{\rho v^2}{2}\right)_2 \quad (23.16)$$

$P_s$  = static pressure  
 $v$  = velocity  
 $\rho$  = density

### Atmosphere Parameters

$$T = 15.04 - 0.00649h \quad (24.17)$$

$$p = 101.29 \left[ \frac{(T + 273.1)}{288.08} \right]^{5.256} \quad (24.18)$$

$$\rho = \frac{p}{0.2869(T + 273.1)} \quad (24.19)$$

$T$  = temperature  
 $h$  = height  
 $p$  = pressure  
 $\rho$  = density

25.0 USCS Soil Classification Chart

